

Fibonacci Day maths for CBSE Class 10: answers

CBSE Math Revision · CBSE Class 10 (Class 9 can try most of it) · 23 November · Teacher answers and notes

No calculator. Where a decimal is wanted, the question gives the value to use.

Teacher notes

- Fibonacci Day is 23 November because 11/23, written month first, reads 1, 1, 2, 3.
- The number trick in task A (b) is a good opener: ask a student for two numbers, write ten terms and give the total before they finish adding.
- Task B (a) shows why consecutive Fibonacci numbers make Euclid's algorithm as slow as possible: every quotient is 1.

Starter

1. **13, 21, 34**

- $5 + 8 = 13$, $8 + 13 = 21$, $13 + 21 = 34$

2. **1**

- $21 = 3 \times 7$ and $34 = 2 \times 17$ have no common factor except 1.

3. **$x = (1 \pm \sqrt{5})/2$**

- $x = (1 \pm \sqrt{(1 + 4)})/2 = (1 \pm \sqrt{5})/2$

4. **33, which is one less than 34**

- $1 + 1 + 2 + 3 + 5 + 8 + 13 = 33$
- The 9th term is 34.

Task A: Fibonacci-type puzzles

a. **115**

- 3, 7, 10, 17, 27, 44, 71, 115

b. **$55a + 88b = 11(5a + 8b)$; 781**

- The terms are a , b , $a + b$, $a + 2b$, $2a + 3b$, $3a + 5b$, $5a + 8b$, $8a + 13b$, $13a + 21b$, $21a + 34b$.
- They add to $55a + 88b = 11(5a + 8b)$, and $5a + 8b$ is the 7th term.
- For 3, 7, ...: the 7th term is 71, so the sum is $11 \times 71 = 781$.

c. **5 and 7**

- 3rd term $a + b = 12$; 6th term $3a + 5b = 50$.
- Multiply the first by 3: $3a + 3b = 36$. Subtract: $2b = 14$, so $b = 7$ and $a = 5$.

Task B: Euclid's division algorithm

a. **HCF = 1, in 10 steps; the remainders are the Fibonacci numbers going down**

- $144 = 89 \times 1 + 55$, $89 = 55 \times 1 + 34$, $55 = 34 \times 1 + 21$, $34 = 21 \times 1 + 13$, $21 = 13 \times 1 + 8$,
- $13 = 8 \times 1 + 5$, $8 = 5 \times 1 + 3$, $5 = 3 \times 1 + 2$, $3 = 2 \times 1 + 1$, $2 = 1 \times 2 + 0$
- 10 steps, and the last non-zero remainder is 1.

b. **3, which is the 4th Fibonacci number, and $\text{HCF}(12, 8) = 4$**

- $144 = 21 \times 6 + 18$, $21 = 18 \times 1 + 3$, $18 = 3 \times 6 + 0$, so the HCF is 3.
- 3 is the 4th Fibonacci number, and $4 = \text{HCF}(12, 8)$.

c. **714**

- $\text{HCF} = 1$, so $\text{LCM} = 21 \times 34 = 714$.

Task C: The golden ratio

a. **$\phi = (1 + \sqrt{5})/2 = 1.618$**

- $x^2 - x - 1 = 0$ gives $x = (1 \pm \sqrt{5})/2$; the positive root is $(1 + \sqrt{5})/2$.
- $(1 + 2.236)/2 = 3.236/2 = 1.618$

b. **Both are $(3 + \sqrt{5})/2$**

- $\phi^2 = (1 + 2\sqrt{5} + 5)/4 = (6 + 2\sqrt{5})/4 = (3 + \sqrt{5})/2$
- $\phi + 1 = (1 + \sqrt{5} + 2)/2 = (3 + \sqrt{5})/2$

c. **1.619 and 1.618: very close to ϕ**

- $34 \div 21 = 1.6190\dots$
- $55 \div 34 = 1.6176\dots$, which is 1.618 to 3 d.p.

Extension

1. **89**

- Your last move is 1 step or 2 steps, so $\text{ways}(n) = \text{ways}(n - 1) + \text{ways}(n - 2)$.
- 1, 2, 3, 5, 8, 13, 21, 34, 55, 89

2. **34**

- The left end is one upright domino (leaving 2 by 7) or two flat dominoes (leaving 2 by 6), so $\text{ways}(n) = \text{ways}(n - 1) + \text{ways}(n - 2)$.
- 1, 2, 3, 5, 8, 13, 21, 34 for $n = 1$ to 8.

Every question is original and every answer was recomputed by computer before publishing.