

Vector Algebra

CBSE Class 12 Maths · Knowledge organiser

KEY DEFINITIONS

Vector

A quantity with magnitude and direction.

Scalar product

$a \cdot b = |a||b|\cos \theta$, a number; zero for perpendicular vectors.

Vector product

$a \times b$, a vector perpendicular to both, of magnitude $|a||b|\sin \theta$.

YOU SHOULD BE ABLE TO...

- Find magnitudes, unit vectors, direction cosines, section-formula points and simple dot/cross products.
- Use dot and cross products for angles, projections, perpendicularity, collinearity and areas at 2- and 3-mark level.
- Write complete multi-step vector solutions (triangle and parallelogram problems, case studies with forces or displacements).
- Work with vectors given only by magnitudes and angles, minimisation and perpendicular-component arguments.

COMMON MISTAKES

- ✗ Middle term of the cross-product determinant: it is $-\hat{j}(a_1b_3 - a_3b_1)$.
- ✗ Using $\frac{m\vec{a}+n\vec{b}}{m+n}$ (weights swapped) in the section formula.
- ✗ Giving only one sign for a unit perpendicular vector, or the wrong one when a condition (acute/obtuse with an axis) is given.
- ✗ Confusing the scalar projection (a number) with the projection vector.

KEY FORMULAS

Magnitude; unit vector $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$, $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

Direction cosines $l = \frac{x}{|\vec{r}|}$, $m = \frac{y}{|\vec{r}|}$, $n = \frac{z}{|\vec{r}|}$, $l^2 + m^2 + n^2 = 1$

PQ $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Section, $m : n$

internal; $\frac{m\vec{b}+n\vec{a}}{m+n}$; $\frac{m\vec{b}-n\vec{a}}{m-n}$
external

Dot product $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos \theta = a_1b_1 + a_2b_2 + a_3b_3$

Perpendicular;
projection of $\vec{a}$ on $\vec{b}$ $\vec{a} \cdot \vec{b} = 0$; $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

Cross product $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin \theta \hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Area:

parallelogram; $|\vec{a} \times \vec{b}|$; $\frac{1}{2}|\vec{a} \times \vec{b}|$
triangle

Parallelogram

from its diagonals $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

Parallel $\vec{a} \times \vec{b} = \vec{0}$; $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$; $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

WORKED EXAMPLE

$|\vec{a}| = 2$, $|\vec{b}| = 3$, $\vec{a} \cdot \vec{b} = 3$: $|\vec{a} - \vec{b}|^2 = 4 + 9 - 6 = 7$, so
 $|\vec{a} - \vec{b}| = \sqrt{7}$; $\cos \theta = \frac{1}{2}$, $\theta = \frac{\pi}{3}$.