

# Three Dimensional Geometry

CBSE Class 12 Maths · Knowledge organiser

## KEY DEFINITIONS

### Direction cosines

The cosines of the angles a line makes with the x-, y- and z-axes;  $l^2 + m^2 + n^2 = 1$ .

### Direction ratios

Any numbers proportional to the direction cosines.

### Skew lines

Lines in space that are neither parallel nor intersecting.

## YOU SHOULD BE ABLE TO...

- Find direction cosines/ratios and write vector and Cartesian equations of a line through a point or two points.
- Convert between forms, find angles between lines, test parallel/perpendicular/coincident lines and distances between parallel lines.
- Score full marks on 5-mark shortest-distance, intersection and foot-of-perpendicular/image questions and on 3-D case studies.
- Solve common-perpendicular, image-of-a-point and multi-part triangle problems in 3-D.

## COMMON MISTAKES

- ✗ Reading direction ratios before putting the line in standard form (signs and coefficients of  $x, y, z$ ).
- ✗ Omitting the modulus in the angle formula (the angle between lines is taken acute) or in the distance formula.
- ✗ Using the skew-line formula for parallel lines (it gives  $0/0$ ).
- ✗ Declaring lines intersecting after matching only two coordinates — always check the third.

## KEY FORMULAS

DCs from

direction ratios  $l = \frac{a}{\sqrt{a^2+b^2+c^2}}, \dots$   
 $a, b, c$

Line through  $\vec{a}$

parallel to  $\vec{b}$

$$\vec{r} = \vec{a} + \lambda \vec{b}; \quad \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Through two points

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}); \quad \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Angle between lines

$$\cos \theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Perpendicular; parallel

$$a_1a_2 + b_1b_2 + c_1c_2 = 0; \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Shortest

distance, skew lines

$$d = \frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Parallel lines

$$d = \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}$$

Lines intersect  $\Leftrightarrow$  shortest distance = 0. Foot of perpendicular from  $P$ : general point  $Q$  on the line with  $\vec{PQ} \cdot \vec{b} = 0$

## WORKED EXAMPLE

Line through  $(1, 0, -1)$  and  $(3, 2, 0)$ :  $\frac{x-1}{2} = \frac{y}{2} = \frac{z+1}{1}$ ;  
direction cosines  $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ .