

Relations and Functions

CBSE Class 12 Maths · Knowledge organiser

KEY DEFINITIONS

Reflexive, symmetric, transitive

The three properties that together make an equivalence relation.

One-one function

Different inputs always give different outputs.

Onto function

Every element of the codomain is the image of some element of the domain.

KEY FORMULAS

Reflexive $(a, a) \in R \forall a$; symmetric $(a, b) \in R \Rightarrow (b, a) \in R$; transitive $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$; equivalence = all three

One-one: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Onto: range = codomain. Bijective: both

Counting, $|A| = m, |B| = n$ relations on $A : 2^{m^2}$; functions $A \rightarrow B : n^m$

One-one $A \rightarrow B$ ($m \leq n$); bijections ($m = n$) $\frac{n!}{(n-m)!}; n!$

YOU SHOULD BE ABLE TO...

- Test a relation for reflexive, symmetric and transitive and tell whether a function is one-one or onto.
- Prove equivalence relations, write equivalence classes, count relations and functions, and prove one-one or onto properly.
- Write the 5-mark relation and function proofs in full, with a heading and reason for each property and counter-examples where needed.
- Handle bijection proofs on restricted domains, tricky counting of relations and the hardest assertion-reason items.

WORKED EXAMPLE

On $\{1, \dots, 6\}$, $a R b \iff 3 \mid (a - b)$: classes $\{1, 4\}, \{2, 5\}, \{3, 6\}$.

COMMON MISTAKES

- ✗ Checking reflexivity for only the elements that appear in R instead of all of A .
- ✗ Showing onto by solving for x but not checking that x lies in the domain (e.g. $x \in \mathbb{N}, x \neq 2$).
- ✗ Forgetting that the answer depends on the domain and codomain: x^2 is not one-one on $\mathbb{R}$ but is one-one on $[0, \infty)$.
- ✗ Giving an “example” instead of a proof when asked to show a property holds; a single counter-example is enough only to show it fails.