

Inverse Trigonometric Functions

CBSE Class 12 Maths · Knowledge organiser

KEY DEFINITIONS

Principal value branch

The range chosen so that an inverse trigonometric function gives exactly one value.

Inverse sine

$\sin^{-1}x$ is the angle in $[-\pi/2, \pi/2]$ whose sine is x .

Domain restriction

Trig functions are made one-one by restricting their domains before inverting.

KEY FORMULAS

Principal ranges	$\sin^{-1} : [-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$,
	$\cos^{-1} : [-1, 1] \rightarrow [0, \pi]$,
	$\tan^{-1} : \mathbb{R} \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$
More ranges	$\cot^{-1} : \mathbb{R} \rightarrow (0, \pi)$, $\sec^{-1} : [0, \pi] - \{\frac{\pi}{2}\}$,
	$\operatorname{cosec}^{-1} : [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
$\sec^{-1}$, $\operatorname{cosec}^{-1}$ domain	$\mathbb{R} - (-1, 1)$
$\sin(\sin^{-1}x) = x$ for $x \in [-1, 1]$; $\sin^{-1}(\sin x) = x$ only for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ (similarly for the others)	

YOU SHOULD BE ABLE TO...

- Write the principal value branch of every inverse trig function and find principal values of standard values.
- Find domains, simplify compositions like $\sin^{-1}(\sin x)$ using the principal range and evaluate expressions such as $\cos(\tan^{-1} \frac{3}{4})$.
- Write complete multi-part answers (the case studies, the 5-mark simplification and the multi-part domain questions), stating the branch you use at each step.
- Handle radian inputs like $\sin^{-1}(\sin 3)$, substitution-based simplifications and justifying the signs when you write $\sin \theta$ and $\cos \theta$ in terms of x .

WORKED EXAMPLE

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}, \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}.$$

COMMON MISTAKES

- ✗ Writing $\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \frac{2\pi}{3}$ — the answer must lie in the principal range.
- ✗ Giving $\cos^{-1}(-x)$ a negative value: $\cos^{-1}$ is never negative.
- ✗ Mixing up ranges of $\cot^{-1}(0, \pi)$ and $\tan^{-1}(-\frac{\pi}{2}, \frac{\pi}{2})$.
- ✗ Ignoring the sign of a square root after a substitution (e.g. $\sqrt{\cos^2 \theta} = |\cos \theta|$).