

Integrals

CBSE Class 12 Maths · Knowledge organiser

KEY DEFINITIONS

Antiderivative

F is an antiderivative of f when $F' = f$; add a constant C.

Definite integral

$$\int_a^b f(x) dx = F(b) - F(a).$$

Integration by parts

The rule for integrating a product, choosing u in the order ILATE.

YOU SHOULD BE ABLE TO...

- Integrate the standard forms, use simple substitutions and evaluate basic definite integrals.
- Use the special-form integrals, partial fractions and integration by parts on board-standard questions.
- Write complete long answers on definite integrals, including the property questions, with each step and the property named.
- Handle the hardest property-based integrals and the tricky substitutions that decide 95+.

COMMON MISTAKES

- ✗ Forgetting $+C$ in indefinite integrals, or keeping old limits after a substitution.
- ✗ $\int \frac{dx}{2x+3} = \frac{1}{2} \log |2x+3|$, not $\log |2x+3|$.
- ✗ Applying partial fractions to an improper fraction without dividing first.
- ✗ Using $F(b) - F(a)$ when f is discontinuous in $[a, b]$.

KEY FORMULAS

Basic	$\int x^n dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1),$ $\int \frac{1}{x} dx = \log x + C$
Exp	$\int e^x dx = e^x + C, \quad \int a^x dx = \frac{a^x}{\log a} + C$
Trig	$\int \sin x dx = -\cos x + C,$ $\int \cos x dx = \sin x + C,$ $\int \sec^2 x dx = \tan x + C$
Trig	$\int \operatorname{cosec}^2 x dx = -\cot x + C,$ $\int \sec x \tan x dx = \sec x + C,$ $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$
Trig	$\int \tan x dx = \log \sec x + C,$ $\int \cot x dx = \log \sin x + C$
sec, cosec	$\int \sec x dx = \log \sec x + \tan x + C,$ $\int \operatorname{cosec} x dx = \log \operatorname{cosec} x - \cot x + C$
Inverse trig	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C,$ $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
Special	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \left \frac{x-a}{x+a} \right + C,$ $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left \frac{a+x}{a-x} \right + C$
Special	$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C,$ $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$
Special	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log \left x + \sqrt{x^2 \pm a^2} \right + C$
Root	$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$
Root	$\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2}$ $\pm \frac{a^2}{2} \log \left x + \sqrt{x^2 \pm a^2} \right + C$
By parts (ILATE for the first function u)	$\int uv dx = u \int v dx - \int (u' \int v dx) dx$
Special form	$\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$
$ax^2 + bx + c$: complete the square; for $\frac{px+q}{ax^2+bx+c}$ write $px+q = A \frac{d}{dx}(ax^2+bx+c) + B$	
Partial fractions	$\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b},$ $\frac{\dots}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$

More on the full formula sheet.

WORKED EXAMPLE

$$\int \frac{3x+1}{x^2+2x+5} dx: 3x+1 = \frac{3}{2}(2x+2) - 2, \text{ giving}$$
$$\frac{3}{2} \log(x^2+2x+5) - \tan^{-1} \frac{x+1}{2} + C.$$