

Determinants

CBSE Class 12 Maths · Knowledge organiser

KEY DEFINITIONS

Determinant

A number found from a square matrix; for a 2×2 matrix it is $ad - bc$.

Adjoint

The transpose of the matrix of cofactors.

Inverse

$A^{-1} = \text{adj } A / |A|$, which exists only when $|A| \neq 0$.

YOU SHOULD BE ABLE TO...

- Expand 2×2 and 3×3 determinants, find minors and cofactors, and find inverses of 2×2 matrices.
- Use determinants for area, collinearity and lines; use $|kA|$, $|\text{adj } A|$ and inverse results; decide consistency of small systems.
- Score full marks on the 5-mark matrix-method question and on determinant case studies, with every step mark visible.
- Handle parameter systems (unique / none / infinitely many), adj-inverse identities and unfamiliar applications.

COMMON MISTAKES

- ✗ Sign pattern of cofactors $\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$ forgotten, especially for $A_{12}, A_{21}, A_{23}, A_{32}$.
- ✗ Writing the matrix of cofactors instead of its transpose as $\text{adj } A$.
- ✗ $|kA| = k|A|$ instead of $k^n|A|$.
- ✗ Forgetting the modulus (or one of the two cases) in area problems.

KEY FORMULAS

$$2 \times 2 \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\text{Minor; cofactor} \quad M_{ij}; \quad A_{ij} = (-1)^{i+j} M_{ij}$$

$|A| = \sum_j a_{ij} A_{ij}$ along any row (or column); elements $\times$ cofactors of a different row sum to 0

$$\text{Area of triangle} \quad \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}; \quad \text{collinear} \iff \det = 0$$

Adjoint =

transpose of the cofactor matrix $A(\text{adj } A) = (\text{adj } A)A = |A|I$

$$\text{Inverse} \quad A^{-1} = \frac{1}{|A|} \text{adj } A \quad (|A| \neq 0)$$

$$\text{Order } n \quad |AB| = |A||B|, \quad |A^T| = |A|, \quad |kA| = k^n|A|, \\ |\text{adj } A| = |A|^{n-1}, \quad |A^{-1}| = \frac{1}{|A|}$$

$$AX = B \quad |A| \neq 0 : X = A^{-1}B \text{ (unique)}$$

$|A| = 0$: $(\text{adj } A)B \neq O \Rightarrow$ inconsistent; $(\text{adj } A)B = O \Rightarrow$ infinitely many or none

WORKED EXAMPLE

Area of the triangle with vertices $(0, 0)$, $(4, 1)$, $(1, 3)$:

$$\frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 4 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = \frac{1}{2}(12 - 1) = \frac{11}{2} \text{ sq units.}$$