

Continuity and Differentiability

CBSE Class 12 Maths · Knowledge organiser

KEY DEFINITIONS

Continuous function

f is continuous at c when the limit of $f(x)$ as $x \rightarrow c$ equals $f(c)$.

Differentiable

The derivative exists at the point; differentiable implies continuous.

Chain rule

$d/dx f(g(x)) = f'(g(x)) g'(x)$.

YOU SHOULD BE ABLE TO...

- Check continuity at a point, find k for continuity and differentiate with the chain rule, logs and exponentials.
- Test differentiability, and use implicit, logarithmic, parametric and inverse-trig differentiation on board-standard questions.
- Write long answers and case studies in full, especially 'prove that' second-derivative questions and x^x -type derivatives.
- Handle piecewise functions with two unknowns, $|x|$ and greatest-integer functions and max/min-defined functions.

COMMON MISTAKES

- ✗ Declaring a function differentiable because the two formulas "join up" — continuity is not enough.
- ✗ Using $\log(u + v) = \log u + \log v$ (false). Take logs only of products, quotients and powers.
- ✗ Treating x^x like x^n or a^x : neither rule applies; use logarithms.
- ✗ Forgetting $\frac{dy}{dx}$ on y -terms in implicit differentiation, or the inner derivative in the chain rule.

KEY FORMULAS

Continuous at c $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$

Differentiable at c : LHD = RHD; differentiable $\Rightarrow$ continuous (not conversely, e.g. $|x|$ at 0)

Standard	$(x^n)' = nx^{n-1}$, $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \sec^2 x$
More trig	$(\cot x)' = -\operatorname{cosec}^2 x$, $(\sec x)' = \sec x \tan x$, $(\operatorname{cosec} x)' = -\operatorname{cosec} x \cot x$
Exp, log	$(e^x)' = e^x$, $(a^x)' = a^x \log a$, $(\log x)' = \frac{1}{x}$, $(\log_a x)' = \frac{1}{x \log a}$
Inverse trig	$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$, $(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$, $(\tan^{-1} x)' = \frac{1}{1+x^2}$
Rules	$(uv)' = u'v + uv'$, $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$
Logarithmic, $y = u^v$	$\log y = v \log u \Rightarrow \frac{1}{y} \frac{dy}{dx} = v' \log u + \frac{vu'}{u}$
Parametric	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}$
Implicit: differentiate term by term,	$\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$

WORKED EXAMPLE

Is $f(x) = \begin{cases} 3x - 2, & x \leq 2 \\ x^2, & x > 2 \end{cases}$ differentiable at 2? $f(2) = 4$,
right limit 4: continuous. LHD = 3, RHD = 4: not differentiable.