

Application of Derivatives

CBSE Class 12 Maths · Knowledge organiser

KEY DEFINITIONS

Increasing function

$f'(x) > 0$ on an interval means f is strictly increasing there.

Critical point

A point where $f'(c) = 0$ or $f'(c)$ does not exist.

Local maximum

A point higher than all nearby points: f' changes from $+$ to $-$.

KEY FORMULAS

Rate of change $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

$f'(x) > 0$ on (a, b) : strictly increasing; $f'(x) < 0$: strictly decreasing

Critical point: $f'(c) = 0$ or $f'(c)$ undefined

First derivative test: f' changes $+$ $\rightarrow$ $-$ (max), $-$ $\rightarrow$ $+$ (min), no change (neither)

Second derivative test $f'(c) = 0$: $f''(c) < 0$ max, $f''(c) > 0$ min, $f''(c) = 0$ test fails

Absolute max/min on $[a, b]$: compare f at critical points and at a, b

YOU SHOULD BE ABLE TO...

- Find a rate of change, find where a function increases or decreases and locate turning points.
- Classify local maxima and minima with a derivative test and find absolute extrema on a closed interval.
- Write full-marks optimisation and case-study answers: function set up, derivative, test, and the value that was asked for.
- Handle monotonicity with parameters or trig functions and the harder optimisation problems.

WORKED EXAMPLE

A spherical balloon is inflated at $100 \text{ cm}^3/\text{s}$. When $r = 5$ cm: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{100}{100\pi} = \frac{1}{\pi} \text{ cm/s}$.

COMMON MISTAKES

- ✗ Substituting the particular value before differentiating in a rate problem.
- ✗ Forgetting end points when finding absolute extrema; confusing a local maximum with the absolute maximum.
- ✗ Ignoring points where f' is undefined (e.g. $x^{2/3}$ at 0).
- ✗ Not justifying that the stationary point is a maximum/minimum — this carries a mark.