Integrals Class 12 Important Questions

10 questions · 27 marks · no calculator · step-marked answers

Name _____ Date _____ Score / 27

- Original questions in the board paper's question types, from 1-mark MCQs to a case study. Try every question before you look at the answers.
- No calculator: as in the CBSE board exam, everything works by hand.
- Worked answers with step marks for 5 questions start after the questions. The other 5 open free on the chapter page with a free account (no card): the link is next to each one.
- Online, with more practice: cbsemathrevision.com/class-12/integrals

Section A · Multiple choice and assertion–reason (1 mark each)

1. **1 mark · Integration by substitution**

Which of these is $\int \frac{\log x}{x}\,dx$ for $x > 0$?

(a) $\frac{(\log x)^2}{2} + C$

(b) $\log|\log x| + C$

(c) $\frac{1}{x} + C$

(d) $x \log x - x + C$

2. **1 mark · Definite integrals and their properties**

The value of $\int_0^{\pi/4} \sec^2 x\,dx$ is

(a) 0

(b) 1

(c) $\sqrt{2}$

(d) $\frac{\pi}{4}$

3. 1 mark · Definite integrals and their properties

In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

Assertion (A): $\int_{-1}^{1} x^3\,dx = 0$.

Reason (R): For every continuous function f, $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Section B · Short answer (2 marks each)

4. 2 marks · Integration by substitution

Evaluate $\int \dfrac{2x}{x^2+4}\,dx$.

5. 2 marks · Integration by parts

Evaluate $\int x\cos 3x\,dx$.

Section C · Short answer (3 marks each)

6. **3 marks · Partial fractions**

Evaluate $\displaystyle\int \frac{3x+1}{(x-1)(x+3)}\,dx$.

7. **3 marks · Integration by substitution**

Evaluate $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$.

Section D · Long answer (5 marks each)

8. **5 marks · Definite integrals and their properties**

Evaluate $\displaystyle\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx$.

9. **5 marks · Partial fractions**

Evaluate $\displaystyle\int \frac{x^2 + x + 1}{(x+1)^2(x+2)}\,dx$.

Section E · Case study (4 marks)

10. **4 marks · Definite integrals and their properties**

Cycle on a straight track. A cyclist moves along a straight track with velocity $v(t) = 3t^2 + 2t$ m/s, t seconds after starting.

i. Write $\displaystyle\int v(t)\,dt$. *[1 mark]*

ii. How far does the cyclist travel in the first 2 seconds? *[1 mark]*

iii. The cyclist starts 5 m from a marker. Find the distance from the marker at $t = 3$ and the average velocity over the first 3 seconds. *[2 marks]*

Answers and worked solutions

Question 1 — 1 mark

Put $t = \log x$, $dt = \dfrac{dx}{x}$:

$$\int t\,dt = \frac{t^2}{2} + C = \frac{(\log x)^2}{2} + C$$ **A1**

Correct option: (a) $\dfrac{(\log x)^2}{2} + C$

Question 2 — 1 mark

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/integrals#iq-2

Question 3 — 1 mark

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/integrals#iq-3

Question 4 — 2 marks

Answer: $\log(x^2+4) + C$

Put $t = x^2 + 4$, $dt = 2x\,dx$: $\displaystyle\int \frac{dt}{t}$ **M1**

$= \log(x^2+4) + C$ **A1**

Question 5 — 2 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/integrals#iq-5

Question 6 — 3 marks

Answer: $\log|x-1| + 2\log|x+3| + C$

$\dfrac{3x+1}{(x-1)(x+3)} = \dfrac{A}{x-1} + \dfrac{B}{x+3}$, so

$3x + 1 = A(x+3) + B(x-1)$ **M1**

$x = 1$: $4 = 4A \Rightarrow A = 1$; $x = -3$:

$-8 = -4B \Rightarrow B = 2$ **A1**

Integral $= \log|x-1| + 2\log|x+3| + C$ **A1**

Question 7 — 3 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/integrals#iq-7

Question 8 — 5 marks

Answer: 1

Use $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ with $a+b=4$ **M1**

$$I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}\,dx$$ **A1**

Adding the two forms:

$$2I = \int_1^3 \frac{\sqrt{x}+\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}\,dx = \int_1^3 1\,dx$$ **M1**

$2I = 3 - 1 = 2$ **A1**

$I = 1$ **A1**

Question 9 — 5 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/integrals#iq-9

Question 10 — 4 marks

i. $t^3 + t^2 + C$ **A1**

ii. $[t^3 + t^2]_0^2 = 8 + 4 = 12\ m$ **A1**

iii. $s(3) = 5 + [t^3 + t^2]_0^3 = 5 + 36 = 41\ m$ **M1**

Average velocity $= \dfrac{36}{3} = 12\ \text{m/s}$ **A1**

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