Continuity and Differentiability Class 12 Important Questions

10 questions · 27 marks · no calculator · step-marked answers

Name _____ Date _____ Score / 27

- Original questions in the board paper's question types, from 1-mark MCQs to a case study. Try every question before you look at the answers.
- No calculator: as in the CBSE board exam, everything works by hand.
- Worked answers with step marks for 5 questions start after the questions. The other 5 open free on the chapter page with a free account (no card): the link is next to each one.
- Online, with more practice: cbsemathrevision.com/class-12/continuity-and-differentiability

Section A · Multiple choice and assertion–reason (1 mark each)

1. 1 mark · Continuity

For which value of k is $f(x) = kx - 3$ (for $x \le 4$), $f(x) = x + 5$ (for $x > 4$) continuous at $x = 4$?

(a) 3 (b) 2

(c) $\frac{9}{4}$ (d) 4

2. 1 mark · Differentiability and derivatives

The derivative of $\sin(\log x)$, for $x > 0$, is

(a) $\frac{\cos(\log x)}{x}$ (b) $\cos(\log x)$

(c) $-\frac{\cos(\log x)}{x}$ (d) $\frac{\sin(\log x)}{x}$

3. 1 mark · Differentiability and derivatives

In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

Assertion (A): $f(x) = |x - 3|$ is continuous at $x = 3$ but not differentiable there.

Reason (R): Every continuous function is differentiable.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Section B · Short answer (2 marks each)

4. **2 marks · Implicit and parametric differentiation**

The curve $x^2 - xy + 2y^2 = 8$ defines y implicitly as a function of x. Find $\dfrac{dy}{dx}$.

5. **2 marks · Differentiability and derivatives**

Use the product rule to find the derivative of $e^{x^2}\cos x$.

Section C · Short answer (3 marks each)

6. **3 marks · Implicit and parametric differentiation**

If $x = 2t^2$ and $y = t^3$, find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ in terms of t, and their values at $t = 2$.

7. **3 marks · Logarithmic differentiation**

Find $\dfrac{dy}{dx}$ if $y = (x^2+1)^x$.

Section D · Long answer (5 marks each)

8. **5 marks · Second-order derivatives**

Given $y = e^{3x}(a\cos 2x + b\sin 2x)$ for constants a and b, prove that $y'' - 6y' + 13y = 0$.

9. **5 marks · Continuity**

$f(x) = \begin{cases} 3x + a, & x < 1 \\ x^2 + bx, & 1 \le x \le 3 \\ 12, & x > 3 \end{cases}$ is continuous for all real x. (a) Find a and b. (b) Is f differentiable at $x = 1$? At $x = 3$?

Section E · Case study (4 marks)

10. **4 marks · Continuity**

Taxi fares. A taxi charges a flat ₹50 for the first 2 km and ₹15 for each km after that, so the fare for x km is $f(x) = 50$ for $0 < x \le 2$ and $f(x) = 50 + 15(x - 2)$ for $x > 2$.

i. Is f continuous at $x = 2$? *[1 mark]*

ii. Is f differentiable at $x = 2$? *[1 mark]*

iii. Find the fare for a 10 km ride and $f'(6)$. *[2 marks]*

Answers and worked solutions

Question 1 — 1 mark

$f(4) = 4k - 3$ and the right-hand limit is $4 + 5 = 9$: $4k - 3 = 9 \Rightarrow k = 3$ **A1**

Correct option: (a) 3

Question 2 — 1 mark

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/continuity-and-differentiability#iq-2

Question 3 — 1 mark

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/continuity-and-differentiability#iq-3

Question 4 — 2 marks

Answer: $\dfrac{y - 2x}{4y - x}$

Differentiate: $2x - y - x\dfrac{dy}{dx} + 4y\dfrac{dy}{dx} = 0$ **M1**

$\dfrac{dy}{dx} = \dfrac{y - 2x}{4y - x}$ **A1**

Question 5 — 2 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/continuity-and-differentiability#iq-5

Question 6 — 3 marks

Answer: $\dfrac{3t}{4}$, $\dfrac{3}{16t}$; $\dfrac{3}{2}$ and $\dfrac{3}{32}$

$\dfrac{dx}{dt} = 4t$, $\dfrac{dy}{dt} = 3t^2$, so $\dfrac{dy}{dx} = \dfrac{3t}{4}$ ($= \dfrac{3}{2}$ at $t = 2$) **M1**

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{3t}{4}\right) \div \dfrac{dx}{dt} = \dfrac{3}{4} \times \dfrac{1}{4t} = \dfrac{3}{16t}$ **A1**

At $t = 2$: $\dfrac{3}{32}$ **A1**

Question 7 — 3 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/continuity-and-differentiability#

Question 8 — 5 marks

Answer: Shown

$y' = 3e^{3x}(a\cos 2x + b\sin 2x) + e^{3x}(-2a\sin 2x + 2b\cos 2x)$ **M1**

$= 3y + 2e^{3x}(b\cos 2x - a\sin 2x)$ **A1**

$y'' = 3y' + 6e^{3x}(b\cos 2x - a\sin 2x) + 2e^{3x}(-2b\sin 2x - 2a\cos 2x)$ **M1**

$= 3y' + 3(y' - 3y) - 4y = 6y' - 13y$ **A1**

So $y'' - 6y' + 13y = 0$ **A1**

Question 9 — 5 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/continuity-and-differentiability#

Question 10 — 4 marks

i. $f(2) = 50$ and $\lim_{x \to 2^+}[50 + 15(x - 2)] = 50$: yes **A1**

ii. Left derivative 0, right derivative 15: no **A1**

iii. $f(10) = 50 + 15 \times 8 = ₹170$ **M1**

$f'(6) = 15$ (₹ per km) **A1**

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