Application of Derivatives Class 12 Important Questions

10 questions · 27 marks · no calculator · step-marked answers

Name _____ Date _____ Score / 27

- Original questions in the board paper's question types, from 1-mark MCQs to a case study. Try every question before you look at the answers.
- No calculator: as in the CBSE board exam, everything works by hand.
- Worked answers with step marks for 5 questions start after the questions. The other 5 open free on the chapter page with a free account (no card): the link is next to each one.
- Online, with more practice: cbsemathrevision.com/class-12/application-of-derivatives

Section A · Multiple choice and assertion–reason (1 mark each)

1. **1 mark · Increasing and decreasing functions**

$f(x) = x^3 - 3x$ is decreasing on the interval

(a) $(-1, 1)$ (b) $(-\infty, -1)$

(c) $(1, \infty)$ (d) $\mathbb{R}$

2. **1 mark · Rate of change**

How fast does the area of a circle change with its radius r at the instant when $r = 7$ cm?

(a) 14π cm^2 per cm (b) 49π cm^2 per cm

(c) 7π cm^2 per cm (d) 28π cm^2 per cm

3. **1 mark · Maxima and minima**

In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

Assertion (A): $x = 0$ is a point of local minimum of $f(x) = x^3$.

Reason (R): For $f(x) = x^3$, $f'(0) = 0$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Section B · Short answer (2 marks each)

4. **2 marks · Rate of change**

The edge of a cube is increasing at 3 cm/s. How fast is its volume increasing when the edge is 10 cm long?

5. **2 marks · Increasing and decreasing functions**

Find the intervals in which $f(x) = 2x^2 - 8x + 5$ is increasing and those in which it is decreasing.

Section C · Short answer (3 marks each)

6. **3 marks · Maxima and minima**

Find the local maximum and local minimum values of $f(x) = x^3 - 6x^2 + 9x + 2$.

7. **3 marks · Increasing and decreasing functions**

On which intervals is $f(x) = x^4 - 8x^2$ strictly increasing, and on which is it strictly decreasing?

Section D · Long answer (5 marks each)

8. **5 marks · Optimisation problems**

A rectangular garden of area $200\ \text{m}^2$ is to be fenced on three sides; the fourth side is a wall. Find the dimensions that need the least fencing, and that length of fencing.

9. **5 marks · Optimisation problems**

A closed cylindrical can must hold $128\pi\ \text{cm}^3$. Find the radius and height that use the least metal (least total surface area), and that surface area.

Section E · Case study (4 marks)

10. **4 marks · Optimisation problems**

Workshop profit. A workshop's revenue from making x chairs a week is $R(x) = 120x - 2x^2$ and its cost is $C(x) = 20x + 400$ (both in ₹ hundreds), so its profit is $P(x) = R(x) - C(x)$.

i. Find $P'(x)$. *[1 mark]*

ii. How many chairs give the greatest profit? *[1 mark]*

iii. Find the greatest profit and show that it is a maximum. *[2 marks]*

Answers and worked solutions

Question 1 — 1 mark

$f'(x) = 3x^2 - 3 = 3(x-1)(x+1) < 0$ exactly when $-1 < x < 1$ **A1**

Correct option: (a) $(-1, 1)$

Question 2 — 1 mark

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/application-of-derivatives#iq-2

Question 3 — 1 mark

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/application-of-derivatives#iq-3

Question 4 — 2 marks

Answer: $900\ \text{cm}^3/\text{s}$

$V = x^3 \Rightarrow \dfrac{dV}{dt} = 3x^2 \dfrac{dx}{dt}$ **M1**

$= 3 \times 100 \times 3 = 900\ \text{cm}^3/\text{s}$ **A1**

Question 5 — 2 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/application-of-derivatives#iq-5

Question 6 — 3 marks

Answer: Local max 6 at $x = 1$; local min 2 at $x = 3$

$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3) = 0 \Rightarrow x = 1, 3$ **M1**

$f''(x) = 6x - 12$: $f''(1) = -6 < 0$, local maximum $f(1) = 6$ **A1**

$f''(3) = 6 > 0$, local minimum $f(3) = 2$ **A1**

Question 7 — 3 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/application-of-derivatives#iq-

Question 8 — 5 marks

Answer: $10\ m$ by $20\ m$ (the $20\ m$ side parallel to the wall); $40\ m$

Let the two sides at right angles to the wall be $x\ m$ and the side parallel to it $y\ m$: $xy = 200$ **M1**

Fencing $L = 2x + y = 2x + \dfrac{200}{x}$ **A1**

$\dfrac{dL}{dx} = 2 - \dfrac{200}{x^2} = 0 \Rightarrow x^2 = 100 \Rightarrow x = 10$ **M1**

$\dfrac{d^2L}{dx^2} = \dfrac{400}{x^3} > 0$, so this is a minimum **A1**

$x = 10\ m$, $y = 20\ m$; least fencing $= 40\ m$ **A1**

Question 9 — 5 marks

Worked answer: free with a free account (no card) on the chapter page: cbsemathrevision.com/class-12/application-of-derivatives#iq-

Question 10 — 4 marks

i. $P(x) = 100x - 2x^2 - 400$, so $P'(x) = 100 - 4x$ **A1**

ii. $100 - 4x = 0 \Rightarrow x = 25$ **A1**

iii. $P''(x) = -4 < 0$, so $x = 25$ gives a maximum **M1**

$P(25) = 2500 - 1250 - 400 = 850$, i.e. ₹ 85000 **A1**

More on this chapter, free: revision notes, a Route to 95 and practice with instant feedback at cbsemathrevision.com/class-12/application-of-derivatives

Our own original questions, written and checked by CBSE Math Revision. Independent, not affiliated with CBSE or NCERT.