CBSE gives no formula booklet: learn every result on this sheet. Organised by NCERT chapter.

1 Relations & Functions

Reflexive $(a,a)\in R\,\forall a$; symmetric $(a,b)\in R \Rightarrow (b,a)\in R$; transitive $(a,b),(b,c)\in R \Rightarrow (a,c)\in R$; equivalence = all three

One-one: $f(x_1)=f(x_2)\Rightarrow x_1=x_2$. Onto: range = codomain. Bijective: both

Counting, $|A|=m$, $|B|=n$: relations on A : 2^{m^2}; functions $A\to B$: n^m

One-one $A\to B$ ($m\le n$); bijections ($m=n$): $\frac{n!}{(n-m)!}$; $n!$

2 Inverse Trigonometric Functions

Principal ranges: $\sin^{-1}:[-1,1]\to[-\frac{\pi}{2},\frac{\pi}{2}]$, $\cos^{-1}:[-1,1]\to[0,\pi]$, $\tan^{-1}:\mathbb{R}\to(-\frac{\pi}{2},\frac{\pi}{2})$

More ranges: $\cot^{-1}:\mathbb{R}\to(0,\pi)$, $\sec^{-1}:[0,\pi]-\{\frac{\pi}{2}\}$, $\operatorname{cosec}^{-1}:[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$

$\sec^{-1}$, $\operatorname{cosec}^{-1}$ domain: $\mathbb{R}-(-1,1)$

$\sin(\sin^{-1}x)=x$ for $x\in[-1,1]$; $\sin^{-1}(\sin x)=x$ only for $x\in[-\frac{\pi}{2},\frac{\pi}{2}]$ (similarly for the others)

3 Matrices

Product $A_{m\times n}B_{n\times p}$ is $m\times p$; in general $AB\ne BA$; $AB=O$ is possible with $A,B\ne O$

Transpose: $(A^T)^T=A$, $(A+B)^T=A^T+B^T$, $(kA)^T=kA^T$, $(AB)^T=B^TA^T$

Symmetric $A^T=A$; skew-symmetric $A^T=-A$ (diagonal entries 0)

Any square A: $A=\frac{1}{2}(A+A^T)+\frac{1}{2}(A-A^T)$

Inverse: $AB=BA=I\Rightarrow B=A^{-1}$; $(AB)^{-1}=B^{-1}A^{-1}$

4 Determinants

2×2: $\begin{vmatrix} a & b\\ c & d\end{vmatrix}=ad-bc$

Minor; cofactor: M_{ij}; $A_{ij}=(-1)^{i+j}M_{ij}$

$|A|=\sum_j a_{ij}A_{ij}$ along any row (or column); elements × cofactors of a different row sum to 0

Area of triangle: $\frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}\right|$; collinear $\iff$ det $=0$

Adjoint = transpose of the cofactor matrix: $A(\operatorname{adj}A)=(\operatorname{adj}A)A=|A|I$

Inverse: $A^{-1}=\frac{1}{|A|}\operatorname{adj}A$ ($|A|\ne 0$)

Order n: $|AB|=|A||B|$, $|A^T|=|A|$, $|kA|=k^n|A|$, $|\operatorname{adj}A|=|A|^{n-1}$, $|A^{-1}|=\frac{1}{|A|}$

$AX=B$: $|A|\ne 0$: $X=A^{-1}B$ (unique)

$|A|=0$: $(\operatorname{adj}A)B\ne O$ ⇒ inconsistent; $(\operatorname{adj}A)B=O$ ⇒ infinitely many or none

5 Continuity & Differentiability

Continuous at c: $\lim_{x\to c^-}f(x)=\lim_{x\to c^+}f(x)=f(c)$

Differentiable at c: LHD = RHD; differentiable ⇒ continuous (not conversely, e.g. $|x|$ at 0)

Standard: $(x^n)'=nx^{n-1}$, $(\sin x)'=\cos x$, $(\cos x)'=-\sin x$, $(\tan x)'=\sec^2 x$

More trig: $(\cot x)'=-\operatorname{cosec}^2 x$, $(\sec x)'=\sec x\tan x$, $(\operatorname{cosec}x)'=-\operatorname{cosec}x\cot x$

Exp, log: $(e^x)'=e^x$, $(a^x)'=a^x\log a$, $(\log x)'=\frac{1}{x}$, $(\log_a x)'=\frac{1}{x\log a}$

Inverse trig: $(\sin^{-1}x)'=\frac{1}{\sqrt{1-x^2}}$, $(\cos^{-1}x)'=-\frac{1}{\sqrt{1-x^2}}$, $(\tan^{-1}x)'=\frac{1}{1+x^2}$

Rules: $(uv)'=u'v+uv'$, $\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$, $\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$

Logarithmic, $y=u^v$: $\log y=v\log u\Rightarrow\frac{1}{y}\frac{dy}{dx}=v'\log u+\frac{vu'}{u}$

Parametric: $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$, $\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}$

Implicit: differentiate term by term, $\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$

Log means natural log (base e) in NCERT.

6 Application of Derivatives

Rate of change: $\frac{dA}{dt}=\frac{dA}{dr}\cdot\frac{dr}{dt}$

$f'(x)>0$ on (a,b): strictly increasing; $f'(x)<0$: strictly decreasing

Critical point: $f'(c)=0$ or $f'(c)$ undefined

First derivative test: f' changes + → − (max), − → + (min), no change (neither)

Second derivative test: $f'(c)=0$: $f''(c)<0$ max, $f''(c)>0$ min, $f''(c)=0$ test fails

Absolute max/min on $[a,b]$: compare f at critical points and at a, b

7 Integrals

Basic: $\int x^n\,dx=\frac{x^{n+1}}{n+1}+C$ ($n\ne -1$), $\int\frac{1}{x}\,dx=\log|x|+C$

Exp: $\int e^x\,dx=e^x+C$, $\int a^x\,dx=\frac{a^x}{\log a}+C$

Trig: $\int\sin x\,dx=-\cos x+C$, $\int\cos x\,dx=\sin x+C$, $\int\sec^2 x\,dx=\tan x+C$

Trig: $\int\operatorname{cosec}^2 x\,dx=-\cot x+C$, $\int\sec x\tan x\,dx=\sec x+C$, $\int\operatorname{cosec}x\cot x\,dx=-\operatorname{cosec}x+C$

Trig: $\int\tan x\,dx=\log|\sec x|+C$, $\int\cot x\,dx=\log|\sin x|+C$

sec, cosec: $\int\sec x\,dx=\log|\sec x+\tan x|+C$, $\int\operatorname{cosec}x\,dx=\log|\operatorname{cosec}x-\cot x|+C$

Inverse trig: $\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x+C$, $\int\frac{dx}{1+x^2}=\tan^{-1}x+C$

Special: $\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+C$, $\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+C$

Special: $\int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$, $\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+C$

Special: $\int\frac{dx}{\sqrt{x^2\pm a^2}}=\log\left|x+\sqrt{x^2\pm a^2}\right|+C$

Root: $\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

Root: $\int\sqrt{x^2\pm a^2}\,dx=\frac{x}{2}\sqrt{x^2\pm a^2}\pm\frac{a^2}{2}\log\left|x+\sqrt{x^2\pm a^2}\right|+C$

By parts (ILATE for the first function u): $\int uv\,dx=u\int v\,dx-\int\left(u'\int v\,dx\right)dx$

Special form: $\int e^x\left[f(x)+f'(x)\right]dx=e^xf(x)+C$

ax^2+bx+c: complete the square; for $\frac{px+q}{ax^2+bx+c}$ write $px+q=A\frac{d}{dx}(ax^2+bx+c)+B$

Partial fractions: $\frac{px+q}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$, $\frac{\cdots}{(x-a)^2(x-b)}=\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$

Quadratic factor: $\frac{\cdots}{(x-a)(x^2+bx+c)}=\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$

Fundamental theorem: $\int_a^b f(x)\,dx=F(b)-F(a)$

Properties: $\int_a^b f=-\int_b^a f$, $\int_a^b f=\int_a^c f+\int_c^b f$, $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$

Properties: $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

Properties: $\int_0^{2a} f\,dx=2\int_0^a f\,dx$ if $f(2a-x)=f(x)$, 0 if $f(2a-x)=-f(x)$

Even / odd: $\int_{-a}^a f\,dx=2\int_0^a f\,dx$ (even), 0 (odd)

8 Application of Integrals

Area: $\int_a^b |y|\,dx$, $\int_c^d |x|\,dy$

Circle $x^2+y^2=a^2$: area πa^2; ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$: area πab

Region between a curve and a line: integrate (upper − lower) between the intersection points

9 Differential Equations

Order = highest derivative; degree = power of it (when polynomial in derivatives). General solution has as many constants as the order

Variables separable: $\int\frac{dy}{g(y)}=\int f(x)\,dx+C$

Homogeneous $\frac{dy}{dx}=F\left(\frac{y}{x}\right)$: $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}$

Linear $\frac{dy}{dx}+Py=Q$: I.F. $=e^{\int P\,dx}$, $y\cdot\text{I.F.}=\int Q\cdot\text{I.F.}\,dx+C$

Linear $\frac{dx}{dy}+P_1x=Q_1$: I.F. $=e^{\int P_1\,dy}$, $x\cdot\text{I.F.}=\int Q_1\cdot\text{I.F.}\,dy+C$

10 Vector Algebra

Magnitude; unit vector: $|\vec{a}|=\sqrt{x^2+y^2+z^2}$, $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$

Direction cosines: $l=\frac{x}{|\vec{r}|}$, $m=\frac{y}{|\vec{r}|}$, $n=\frac{z}{|\vec{r}|}$, $l^2+m^2+n^2=1$

$\overrightarrow{PQ}$: $(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$

Section, $m:n$ internal; external: $\frac{m\vec{b}+n\vec{a}}{m+n}$; $\frac{m\vec{b}-n\vec{a}}{m-n}$

Dot product: $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta=a_1b_1+a_2b_2+a_3b_3$

Perpendicular; projection of $\vec{a}$ on $\vec{b}$: $\vec{a}\cdot\vec{b}=0$; $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Cross product: $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

Area: parallelogram; triangle: $|\vec{a}\times\vec{b}|$; $\frac{1}{2}|\vec{a}\times\vec{b}|$

Parallelogram from its diagonals: $\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$

Parallel $\vec{a}\times\vec{b}=\vec{0}$; $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$; $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{k}\times\hat{i}=\hat{j}$

11 Three Dimensional Geometry

DCs from direction ratios a,b,c: $l=\frac{a}{\sqrt{a^2+b^2+c^2}},\ \ldots$

Line through $\vec{a}$ parallel to $\vec{b}$: $\vec{r}=\vec{a}+\lambda\vec{b}$; $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

Through two points: $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$; $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

Angle between lines: $\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

Perpendicular; parallel: $a_1a_2+b_1b_2+c_1c_2=0$; $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Shortest distance, skew lines: $d=\frac{|(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)|}{|\vec{b}_1\times\vec{b}_2|}$

Parallel lines: $d=\frac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}$

Lines intersect ⇔ shortest distance = 0. Foot of perpendicular from P: general point Q on the line with $\overrightarrow{PQ}\cdot\vec{b}=0$

12 Linear Programming

Maximise/minimise $Z=ax+by$ subject to linear constraints, $x,y\ge 0$

Corner point method: the optimum occurs at a corner of the feasible region. Bounded region: both max and min exist

Unbounded: M is the max only if $ax+by>M$ has no point in the feasible region (use $<m$ for the min)

13 Probability

Conditional: $P(E\mid F)=\frac{P(E\cap F)}{P(F)}$, $P(F)\ne 0$

Properties: $P(S\mid F)=1$, $P(E'\mid F)=1-P(E\mid F)$

Multiplication rule: $P(E\cap F)=P(E)P(F\mid E)=P(F)P(E\mid F)$

Independent: $P(E\cap F)=P(E)P(F)$

Union: $P(E\cup F)=P(E)+P(F)-P(E\cap F)$

Total probability (partition $E_1,\ldots,E_n$): $P(A)=\sum_{j=1}^n P(E_j)P(A\mid E_j)$

Bayes' theorem: $P(E_i\mid A)=\frac{P(E_i)P(A\mid E_i)}{\sum_{j=1}^n P(E_j)P(A\mid E_j)}$

Random variable: $p_i\ge 0$, $\sum p_i=1$, $E(X)=\mu=\sum x_ip_i$